Enrollment No: ______________________ **Exam Seat No:**______________________

C.U.SHAH UNIVERSITY

Summer Examination-2016

Subject Name: Problem Solving - II

Subject Code: 5SC04PBE1 **Branch: M.Sc.(Mathematics)**

Semester: 4 **Date: 12/05/2016** **Time: 2:30 To 5:30** **Marks: 70**

Instructions:

(1) Use of Programmable calculator and any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

SECTION – I

Q-1 **Attempt the following questions** **(07)**

a) Classify the PDE $xyr - (x^2 - y^2)s - xyt + py - qx = 2(x^2 - y^2)$. **(02)**

b) Find complete integral of $z = px + qy + c\sqrt{(1 + p^2 + q^2)}$. **(02)**

c) Solve: $r - 4s + 4t = 0$. **(02)**

d) Find number of generators in a cyclic group of order 10. **(01)**

Q-2 **Attempt all questions** **(14)**

a) Solve: $z(x + y)p + z(x - y)q = x^2 + y^2$. **(05)**

b) Solve: $z\,(z^2 + xy)(px - qy) = x^4$ **(05)**

c) Solve: $2z + p^2 + qy + 2y^2 = 0$ by Charpit's method. **(04)**

OR

Q-2 **Attempt all questions** **(14)**

a) Solve: $\frac{\partial^3 z}{\partial x^3} - 2\frac{\partial^3 z}{\partial x^2 \partial y} = 2e^{2x} + 3x^2 y$. **(05)**

b) Solve: $(D^2 - DD' - 2D'^2 + 2D + 2D')z = e^{2x+3y}$. **(05)**

c) Solve: $p^2 + q^2 = x + y$. **(04)**

Q-3 **Attempt all questions** **(14)**

a) By using the method of separation of variables, solve two dimensional Laplace equation $u_{xx} + u_{yy} = 0$. **(07)**

b) If G is a group, then show that it is abelian if any one of the following conditions satisfied. **(07)**

a) $a^2 = e \ \forall\, a \in G$, where e is identity in G.

b) $(ab)^2 = a^2b^2 \ \forall\, a, b, c \in G$

c) $a^3 = a \quad \forall\, a \in G$

d) $a(ab)^2b = a^3b^3 \quad \forall\, a, b \in G$

e) $a = a^{-1} \quad \forall\, a \in G$

f) $(ab)^{-1} = a^{-1}b^{-1} \quad \forall\, ab \in G.$

OR

Q-3 **Attempt all questions** **(14)**

a) Find a root of the equation $e^{-x} - 10x = 0$ correct up to three decimal places using the False-position method. **(5)**

b) Find a root of the equation $x^3 - 9x + 1 = 0$ correct up to three decimal places using the Bisection method. **(5)**

c) Obtain the modified Newton-Raphson formula for finding the q^{th} root of a positive integer N. Using this formula or otherwise, find an approximate value of the cube root of 29. **(4)**

SECTION – II

Q-4 **Attempt the following questions** **(07)**

a) Find number of cosets of H in G, where $G = (Z, +)$ and $H = (4Z, +)$. **(02)**

b) If $f = (2\ 3)$ and $g = (4\ 5)$ are two permutations of S_5, then find $f \circ g$. **(02)**

c) Define interpolation and extrapolation. **(02)**

d) The Newton-Raphson method has a ______ rate of convergence. **(01)**

(a) linear; (b) super-linear; (c) quadratic; (d) none of these

Q-5 **Attempt all questions** **(14)**

a) If Z_{12} is a ring under addition and multiplication modulo 12, then find it's all prime ideals and maximal ideals. **(05)**

b) Show that the polynomial $x^2 + 1$ is irreducible over Z_7. **(05)**

c) Let G be a group of order 15. Then find the number of Sylow subgroups of G of order 3 and 5. **(04)**

OR

Q-5 **Attempt all questions** **(14)**

a) Show that the set of numbers of the form $a + b\sqrt{2}$, where a, b ∈ Q, is a field with respect to addition and multiplication. **(05)**

b) Let R be the ring of all matrices of order 2×2 over integers and let S be the set of all 2×2 matrices of the form $\begin{pmatrix} a & 0 \\ b & 0 \end{pmatrix}$ where a, b are integers, then prove that S is a left ideal but not right ideal of R. **(05)**

c) Prove that the ring of polynomials over R is a Euclidean ring. **(04)**

Q-6 **Attempt all questions** **(14)**

a) Solve the following system of linear equations by the Gauss-Siedel method. $8x + 2y - 2z = 8$; $x - 8y + 3z = -4$; $2x + y + 9z = 12$. **(5)**

b) Solve the following system of linear equations by the Gauss-Elimination method. $8y + 2z = -7$; $3x + 5y + 2z = 8$; $6x + 2y + 8z = 26$. **(5)**

c) The following table gives the values of x and y: **(4)**

x:	5	7	9	11	13	17
y:	150	392	?	1452	2366	5202.

Find the value of y for x = 9, using the Lagrange's interpolation method.

OR

Q-6 **Attempt all Questions** **(14)**

a) Using the Newton's Divided difference formula, find f(8) and f(15) given: **(5)**

x:	4	5	7	10	11	13
f(x):	48	100	294	900	1210	2028.

b) Solve the ODE $dy/dx = x + y^2$, $y(0) = 0$, at $x = 0.2$ using the Runge-Kutta method of 4^{th} order. Choose $h = 0.2$. **(5)**

c) Solve the ODE $dy/dx = 1 + y^2$, $y(0) = 1$, at $x = 0.2$ using the modified Euler's method. Choose $h = 0.1$. **(4)**